A Generalized Voter Model on Complex Networks

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Abstract We study a generalization of the voter model on complex networks, focusing on the scaling of mean exit time. Previous work has defined the voter model in terms of an initially chosen node and a randomly chosen neighbor, which makes it difficult to disentangle the effects of the stochastic process itself relative to the network structure. We introduce a process with two steps, one that selects a pair of interacting nodes and one that determines the direction of interaction as a function of the degrees of the two nodes and a parameter α which sets the likelihood of the higher degree node giving its state to the other node. Traditional voter model behaviors can be recovered within the model, as well as the invasion process. We find that on a complete bipartite network, the voter model is the fastest process. On a random network with power law degree distribution, we observe two regimes. For modest values of α, exit time is dominated by diffusive drift of the system state, but as the high-degree nodes become more influential, the exit time becomes dominated by frustration effects dependent on the exact topology of the network.

Keywords Voter model · Complex networks · Stochastic processes

1 Introduction

The voter model is an extremely simple stochastic process which has been extensively studied on lattices [1] and, in recent years, on complex networks [2–6]. It is closely related to a variety of models of language evolution [7], ecological dynamics [8], opinion dynamics [9], and epidemic spread [10]. The voter model defines a dynamical process where nodes are each assigned one of two states, $+1$ or -1. Connections are defined on a lattice by nearest neighbors or on an arbitrary network by edges. Each update step consists of selecting a

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pair of nodes and giving the state of one node to the other. In the most frequently studied version of the model, the first node chosen adopts the state of the second. For the traditional voter model on complex networks, a node is chosen second with frequency proportional to its degree, and so its influence is fixed by the selection process. In this paper, we introduce a generalized voter model with a single tunable parameter that allows control of the influence of topology in a manner independent of the selection process. In this generalized model, the probability of a node giving its state to its neighbor is proportional to k^{α}, where k is the node's degree and α is chosen.

We focus our attention on the mean exit time, i.e. the mean time to achieve consensus. It has been frequently seen that voter model dynamics follow a two step relaxation process [4, 11]. The state of the system first decays exponentially to an ensemble average state, then fluctuates diffusively around this value until consensus occurs. We find similar behavior in our model, but also observe a frustrated state that prevents the system from reaching the ensemble average value.

Voter model processes based entirely on selection frequency are denoted either *link update* or *node update* [5]. In link update dynamics, every time step an edge is selected uniformly at random. One of the two nodes at the ends of the link is then chosen randomly to give its state to the other. In node update dynamics, the traditional definition of the voter model on complex networks, a node is selected to adopt the state of a random neighbor. For random networks, the neighbor is more likely to be connected to a high degree node than low degree, so high degree nodes have more influence.

2 Generalized Voter Model

The two processes described above specify both the selection of a pair of interacting nodes and which node adopts the state of the other. We separate this process into two distinct steps to better understand the contribution of each aspect of the process. Where edge update and node update dynamics differ in the following, we attach a superscript $\mathcal{E}$ and $\mathcal{N}$, respectively, to the relevant quantity.

Given a network of size N, each node i has state $s_i = \pm 1$ and degree k_i. We define P_{ij} to be the probability of giving to node i the state of node j during a given time step. There are two independent components of this event: the probability S_{ij} of selecting an edge connecting nodes i and j and the probability $W(k_i, k_j)$ that a node with degree k_j gives its state to a node with degree k_i. Thus,

$$P_{ij} = S_{ij} W(k_i, k_j). \tag{1}$$

The form of $W(k_i, k_j)$ is motivated by comparison with node update dynamics in uncorrelated networks. Under the node update process, the probability of giving node i with degree k_i the state of node j with degree k_j is

$$P_{ij}^{\mathcal{N}} = \frac{A_{ij}}{N k_i} \tag{2}$$

where A_{ij} is an element of the adjacency matrix, which has a value of 1 if the nodes share an edge and 0 otherwise. This can be interpreted as the probability $1/N$ of selecting i times the probability $1/k_i$ of following one particular edge out. If node pairs are selected in this

manner, ignoring direction, a particular pair of nodes i and j can be chosen by either picking i and following an edge to j or by picking j and following an edge to i. This gives

$$S_{ij}^{\mathcal{N}} = \frac{A_{ij}}{N} \left(\frac{k_i + k_j}{k_i k_j} \right). \tag{3}$$

We propose a generalization that includes the standard node update dynamics, which requires $S_{ij}^{\mathcal{N}} W(k_i, k_j) = P_{ij}^{\mathcal{N}}$. The only form of $W(k_i, k_j)$ to do this is

$$W(k_i, k_j) = \frac{k_j}{k_i + k_j}. \tag{4}$$

This form is consistent with our desire that $W(k_i, k_j)$ be a probability. It also suggests a one parameter generalization, which forms the basis for the rest of this paper:

$$W(k_i, k_j, \alpha) = \frac{k_j^{\alpha}}{k_i^{\alpha} + k_j^{\alpha}}. \tag{5}$$

Qualitatively, the parameter α determines how much a node asserts its degree when transmitting its state. For $\alpha > 0$, the higher degree node of a pair is more likely to give its state to the lower degree node, a bias that increases with α. For $\alpha < 0$, the opposite is true. The special case $\alpha = 0$ ignores topology in determining the direction of interaction since $W(k_i, k_j, 0) = 1/2$ always. Node update dynamics occur when node selection determines pairs and $\alpha = 1$. A recently investigated "invasion" dynamic, where a node is picked to give its state to a random neighbor (opposite the traditional model), occurs for node selection and $\alpha = -1$ [12]. If all nodes of the network have the same degree, as in a mean field or lattice topology, then all values of α are equivalent to the traditional voter model. It can also be applied to other methods of pair selection. Traditional edge update dynamics are attained for $S_{ij}^{\mathcal{E}} = 1/M$, where M is the total number of edges, and $\alpha = 0$.

This process describes a situation where there is a particular connection between the behavior of the agents and the underlying network on which they live. For example, if this process were to be thought of in the context of opinion dynamics, a value of $\alpha > 0$ would correspond to an individual preferring to think like those who are more connected than those who are less connected. The forces of influence and accessibility compete, such that a small value of α makes all nodes accessible enough to change state quickly, but limits the influence of any one node. A high value of α makes high degree nodes influential, thus able to order their neighborhoods quickly, but those influential nodes themselves will flip only on rare occasions.

To understand the outcome of this process, we will consider the master equation for an arbitrary network. The probability of a system being in state $\mathbf{s} = \{s_i\}$ at time t is defined to be $P(\mathbf{s}, t)$. Denote by $\mathbf{s}^i$ the state $\mathbf{s}$ where $s_i \mapsto -s_i$ and let S_{ij} be the probability of selecting the edge between nodes i and j, irrespective of update type. For brevity, we write $W(k_i, k_j, \alpha) = W_{ij}^{\alpha}$. The master equation is

$$\frac{d}{dt} P(\mathbf{s}, t) = \sum_{ij} S_{ij} W_{ij}^{\alpha} \left(\frac{1 + s_i s_j}{2} \right) P(\mathbf{s}^i, t)$$

$$- S_{ij} W_{ij}^{\alpha} \left(\frac{1 - s_i s_j}{2} \right) P(\mathbf{s}, t). \tag{6}$$

Let $\langle s_i \rangle$ be the ensemble average probability of node i being in a $+1$ state. The evolution of arbitrary ensemble average functions can be found in a straightforward manner [13], giving

$$\frac{d\langle s_i \rangle}{dt} = \sum_j S_{ij} W_{ij}^\alpha \left(\langle s_i \rangle - \langle s_j \rangle \right). \tag{7}$$

We find a conserved magnetization, ρ^*, by choosing coefficients C_i such that

$$\frac{d\rho^*}{dt} = \sum_i C_i \frac{d}{dt} \langle s_i \rangle = 0. \tag{8}$$

Since S_{ij} is symmetric in i and j, this happens for arbitrary networks only if

$$C_i W_{ij}^\alpha = C_j W_{ji}^\alpha, \tag{9}$$

which implies that $C_i \propto k_i^\alpha$. Normalizing,

$$\rho^* = \frac{\sum_i k^\alpha \langle s_i \rangle}{\sum_i k^\alpha} = \frac{1}{N\mu_\alpha} \sum_i k_i^\alpha \langle s_i \rangle \tag{10}$$

where μ_α is the αth moment of the degree distribution. Note that the ensemble conserved magnetization is independent of the process of selecting node pairs.

3 Bipartite Network

The simplest topology with heterogeneous degrees is the fully connected bipartite network. Such a network is given by two groups of nodes, group A with size a and group B with size b. A node in group A is connected to every node in group B, but none in group A, and vice versa. The degree of nodes in A, k_A, is the size of B, giving $k_A = b$ and similarly $k_B = a$. In this situation all edges are interchangeable, so there is no difference between the two selection processes and we need only to consider the effect of α. Let ρ_a be the concentration of $+1$ opinions in A and ρ_b be the concentration of $+1$ opinions in B. In our model, the special value $\alpha = 1$ is equivalent to the case studied in [4] on the same network and we follow a similar procedure, omitting details that can be found there. From (10), the conserved magnetization is
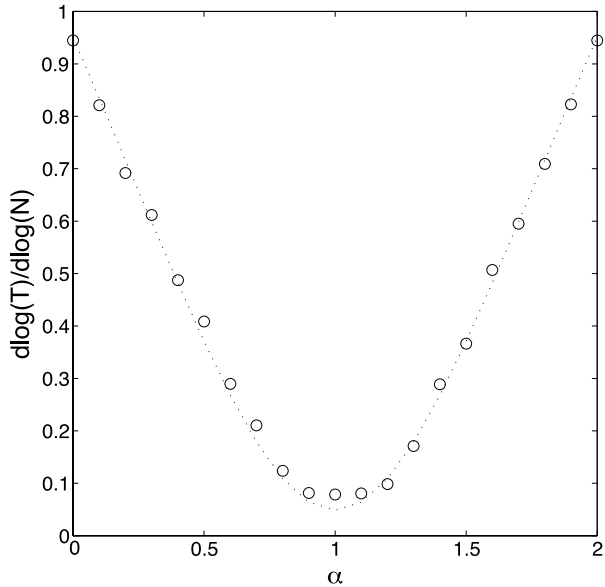
$$\rho^* = \frac{1}{a^{\alpha-1} + b^{\alpha-1}} \left(b^{\alpha-1} \rho_a + a^{\alpha-1} \rho_b \right). \tag{11}$$

For any initial conditions, the ensemble average subgraph densities approaches ρ^*. If all nodes in A start as $+1$ and all nodes in B start as -1, then the probability of ending in the $+1$ state is

$$P_+ = \frac{b^{\alpha-1}}{b^{\alpha-1} + a^{\alpha-1}}. \tag{12}$$

The mean exit time T_α is given by the backward Komologorov equation [14]. T_α solves

Fig. 1 *Circles* are the simulated fit of $d \log T / d \log N$ for a complete bipartite network with groups of size $M = 40$ and N ranging from 100 to 5000. The *dotted line* is the scaling of (16) for the same range of N

$$-\frac{a^\alpha + b^\alpha}{a + b} = (\rho_a - \rho_b)(b^{\alpha-1}\partial_b - a^{\alpha-1}\partial_a)T$$

$$+ \frac{1}{2}(\rho_a + \rho_b - 2\rho_a\rho_b)(a^{\alpha-2}\partial_a^2 + b^{\alpha-2}\partial_b^2)T, \tag{13}$$

where ∂_a and ∂_b are partial derivatives with respect to the initial subgraph densities. The first term describes convection, which brings the subgraph densities to some equal value, and the second term describes the diffusion of the network-wide state [4]. The convective dynamics can be shown to be fast for all α.

The fast step toward equal subgraph densities has a negligible impact on extinction time and we can consider only the subsequent one dimensional problem. We define $\rho = \rho_a = \rho_b$ and apply a change of variables using (11). After integrating,

$$T = -(a^{1-\alpha} + b^{1-\alpha})(a^{\alpha-1} + b^{\alpha-1})$$

$$\times \frac{ab}{a + b}(\rho \log(\rho) + (1 - \rho) \log(1 - \rho)). \tag{14}$$

This has a similar form to the standard voter model, but with a factor that is symmetric about $\alpha = 1$. If we take $a = \lambda b$, then

$$T \propto (2 + \lambda^{1-\alpha} + \lambda^{\alpha-1})\frac{\lambda}{1 + \lambda}b. \tag{15}$$

If $\lambda \gg 1$, corresponding to a star-like graph,

$$T \sim \lambda^{|\alpha-1|}b. \tag{16}$$

This scaling is confirmed in simulations (see Fig. 1). Notably, the standard voter model, $\alpha = 1$, is the fastest process for any complete bipartite network.

4 Random Scale-Free Networks

Similar analysis extends naturally to networks in which a node's degree determines the network structure. Many random network models fall into this category, including any random network generated by the configuration model, like those with scale-free distributions, and Erdos-Renyi networks [15]. Small world networks are not included, however, as certain nodes have exceptional topological characteristics that are independent of their degree [16].

As in (13), we can write the equation satisfied by the mean exit time on an arbitrary network. For the rest of this paper, ρ_l refers to the density of $+1$ states on the subgraph of nodes with degree $k_l = l$. The backwards Komologorov equation now reads

$$-\delta_t = \sum_{lm} S_{lm} W_{lm}^\alpha (\rho_l - \rho_m) \delta_i \partial_l T$$

$$+ \frac{1}{2} \sum_{lm} S_{lm} W_{lm}^\alpha (\rho_l + \rho_m - 2\rho_l \rho_m) \delta_l^2 \partial_l^2 T. \tag{17}$$

There is, in general, a prefactor to the term $\rho_l(1 - \rho_l)$ when evaluating the probability that two nodes have differing states [2]. Numerical simulations show that this is of order one and not generally a constant over time when $\alpha \neq 1$, so it does not affect the final scaling results. The system is again split into a convective term and a diffusive term, however the assumption of fast approach to well-mixed state must be treated more carefully in our generalized model. However, since it was observed in [4] that node update dynamics on a scale free network has fast convection compared to its diffusive exit time, we know that a diffusion-dominated state exists.

We expect that this is not true for all α, though. In the case of $\alpha \gg 1$, a node with degree higher than all its neighbors will act to dictate its neighbors' states, but only rarely be changed itself. The network in this case may not be able to quickly approach the global equilibrium given by ρ^*, since these locally highest degree nodes will be pinned for a time dependent on α. If this duration is longer than the time for the rest of the system to become ordered via drift and diffusion, a quasi-frustrated state occurs. The specific local topology, rather than just degree distributions, dominates the dynamics.

Let us suppose that the exit time is diffusion dominated and then return to discuss the validity of this assumption later. The system can be approximated by a one dimensional equation in $\rho = \frac{1}{\mu_\alpha} \sum_l n_l l^\alpha \rho_l$, where n_l is the fraction of nodes with degree l

$$-N = \frac{1}{\mu_\alpha^2} \left(\sum_{lm} S_{lm} W_{lm}^\alpha l^{2\alpha} \right) \rho(1 - \rho) \partial_\rho^2 T. \tag{18}$$

And thus

$$T \propto \frac{N \mu_\alpha^2}{\sum_{lm} S_{lm} W_{lm}^\alpha l^{2\alpha}}. \tag{19}$$

The denominator can be simplified by noting that

$$\sum_{lm} S_{lm} W_{lm}^\alpha l^{2\alpha} = \frac{1}{2} \left(\sum_{lm} S_{lm} W_{lm}^\alpha l^{2\alpha} + \sum_{lm} S_{lm} W_{ml}^\alpha m^{2\alpha} \right) = \frac{1}{2} \sum_{lm} S_{lm} l^\alpha m^\alpha. \tag{20}$$

This gives:

$$T \propto N \frac{\mu_\alpha^2}{\sum_{lm} S_{lm} l^\alpha m^\alpha}. \tag{21}$$

Since S_{lm} is a probability, the sum can be thought of as a weighted average over selection probabilities. Interestingly, for $\alpha = 0$, neither the form of interaction selection nor the network topology matter. In that case, $\mu_0 = 1$ and $W_{ij} = 1/2$, so

$$T_{\alpha=0} \propto N. \tag{22}$$

This agrees with the observation in [3] that exit times scale with N in situations where the unweighted magnetization is conserved, which corresponds to $\alpha = 0$.

To go farther, we need to specify the selection scheme and the network. We focus our consideration on random uncorrelated scale-free networks with degree distribution $n_k \sim k^{-\nu}$. Networks with power law distributions appear in a variety of social and biological contexts and exhibit a range of interesting behaviors [15]. Let us first consider node update, for which $S_{lm}^{\mathcal{N}} = n_l n_m \frac{l+m}{2\mu_1}$. Then (21) becomes:

$$T^{\mathcal{N}} \propto N \frac{\mu_1 \mu_\alpha}{\mu_{\alpha+1}}. \tag{23}$$

The αth moment can be approximated by an integral:

$$\mu_\alpha \sim \int^{k_{\max}} k^\alpha n(k) dk \tag{24}$$

up to an effective maximum degree $k_{\max}$, defined by $\int_{k_{\max}}^{\infty} n(k) dk = 1/N$ [17]. It is easily seen that $k_{\max} \sim N^{1/(\nu-1)}$.

$$T^{\mathcal{N}} \propto \begin{cases} N^{\frac{\nu-2}{\nu-1}} & \alpha > \nu - 1, \\ N^{\frac{2\nu-\alpha-3}{\nu-1}} & \nu - 2 < \alpha < \nu - 1, \\ N & \alpha < \nu - 2. \end{cases} \tag{25}$$

For $\nu > 2$ and any α, the exit time increases without bound as system size increases. We simulated the process on random network generated by the configuration model [18] and found good agreement with our predictions (see Fig. 2).

For edge update dynamics, $S_{lm}^{\mathcal{E}} = n_l n_m \frac{lm}{\mu_1^2}$. Low degree nodes are selected less frequently under edge selection than node selection. The diffusive exit time can be calculated similarly, giving:

$$T^{\mathcal{E}} \propto N \left(\frac{\mu_1 \mu_\alpha}{\mu_{\alpha+1}} \right)^2. \tag{26}$$

The approximate scaling for edge update is

$$T^{\mathcal{E}} \propto \begin{cases} N^{\frac{\nu-3}{\nu-1}} & \alpha > \nu - 1, \\ N^{\frac{3\nu-2\alpha-5}{\nu-1}} & \nu - 2 < \alpha < \nu - 1, \\ N & \alpha < \nu - 2. \end{cases} \tag{27}$$

This leads to very different scaling behavior. For the parameter regions

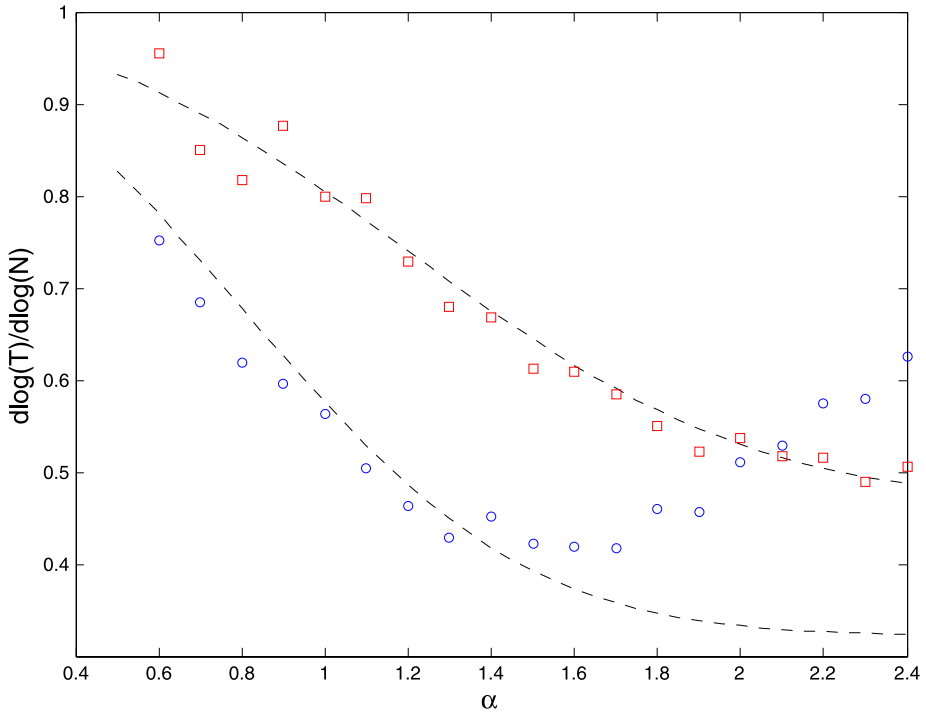$$\nu < 3, \qquad \alpha > \nu - 1, \tag{28}$$

Fig. 2 Simulated values of $d \log T / d \log N$ under node selection based on several hundred runs for N from 750–15000. *Squares* correspond to $\nu = 2.8$, *circles* to $\nu = 2.4$. The *dashed line* is the scaling based on the diffusive estimate, calculated by fitting the numerically calculated sum in (21) for similar values of N. Note that frustration begins to dominate for $\nu = 2.4$ at $\alpha > 1.6$, causing the deviation from the diffusive estimate

$$\nu < \frac{2\alpha + 5}{3}, \qquad \nu - 2 < \alpha < \nu - 1 \qquad (29)$$

the diffusive exit time vanishes as N increases. Simulations show that there is a diffusive region which agrees with our predictions for smaller values of α (see Fig. 3).

The convective process involves an exponential decay of each ρ_i to its ensemble equilibrium value, with rate determined by the network structure. For larger values of α, the state of the highest degree node comes to dominate the value of ρ^*. This causes ρ^* to be either extremely close to 0 or extremely close to 1, so the result of diffusive drift is close to a consensus state. Simulations for this region are consistent with either a very small power law scaling of $T^{\mathcal{E}} \propto N^{0.12}$ or $T^{\mathcal{E}} \propto \log(N)$ (Fig. 3). The latter case would arise if the diffusive time is short compared to the drift time over the range of system sizes tested, consistent with observations (see inset, Fig. 7) of dynamics for these values. Note that this case only is observed under edge selection.

A third regime is apparent when one considers the limit that $\alpha \to \infty$. In this limit, $\rho^* = s_{\max}$, where $s_{\max}$ is the initial state of the highest degree node, either 0 or 1. However, since $W_{ij}^{\infty} = 0$, any node i whose degree is higher than all of its neighbors will take an arbitrarily long time to flip. Such nodes will be called "local leaders," following [19]. The network enters a frustrated state where the convection does not exponentially drive the system to a global state ρ^*. As a result, the time to convergence is dominated by individual nodes and
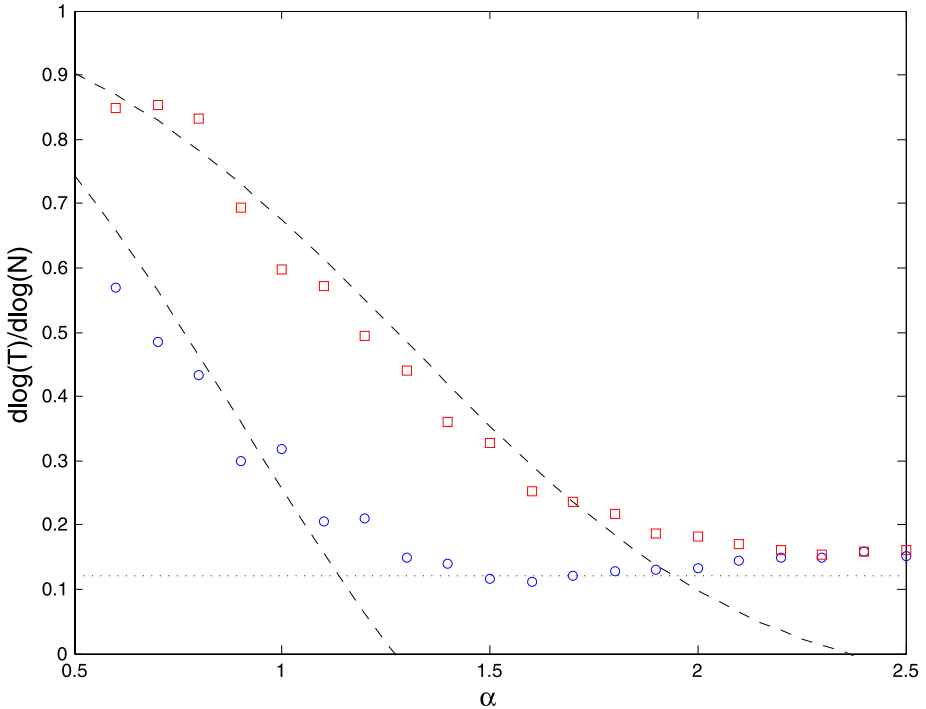
Fig. 3 Simulated values of $d \log T / d \log N$ under edge selection from several hundred runs of values of N from 750–15000. *Square* are for $\nu = 2.8$, *circles* for $\nu = 2.4$. The *dashed line* comes from fitting the numerically calculated sum in (21) for similar values of N. The *horizontal line* is the effective slope of $T \sim \log N$ for the system sizes used

local topology, not global network properties (Fig. 7). An illustration of this is shown for a small network in (Fig. 4).

The continuum treatment in (17) averages over network ensembles before solving for the exit time. In the high α case, considering an averaged network ignores local leader effects and fails to give an accurate solution. The approach to an ensemble equilibrium state does not occur exponentially, so diffusion about this state is not a valid assumption (Fig. 6). Moreover, the state at which the system becomes frustrated differs considerably between individual networks with the same degree distribution and size (inset, Fig. 6).

The degree distribution of locally highest degree nodes can be approximated for the non-assortative case quite simply. The degree distribution of local leaders, $p_{ll}(k)$, is the independent product that a node has degree k and that all k neighbors have a degree less than k [19]:

$$p_{ll}(k) = p(k) \left(\sum_{k' < k} \frac{k' p(k')}{\mu_1} \right)^k . \tag{30}$$

No term is strongly related to system size, so the total number of local leaders N_{ll} scales linearly with N.

The dynamics of these nodes are based on extremely local behavior and thus very hard to approximate, but we can construct the slowest possible node and consider its behavior. For

Fig. 4 An example of node
update dynamics on a small
network containing a global
highest degree node (G) and a
separate local highest degree
node (L) for $\alpha = 1$ and $\alpha = 10$.
The darkness of a node
corresponds to the average
fraction of time spent in a state
opposite the final state of the
network over 1000 realizations
with identical initial conditions.
A darker node has spent more
time in a contrary state than a
light node. For $\alpha = 1$, states are
well mixed and $T = 9.4$. For
$\alpha = 10$, the decay to the
metastable value $\rho*$ does not
occur. The local leader and its
neighborhood spend most of the
time in a contrary state and
$T = 4814$
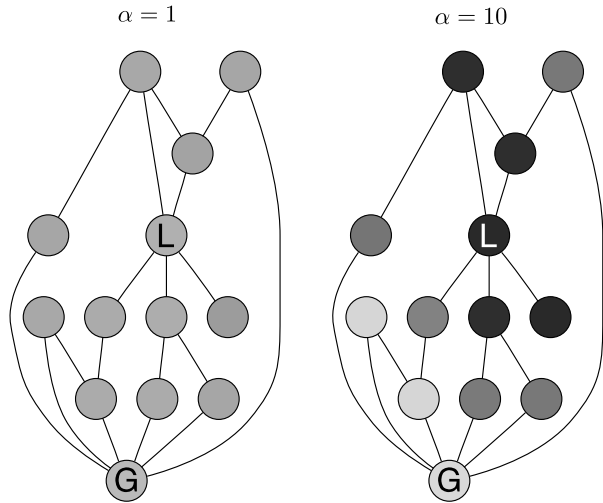
a sufficiently large system, one such node will likely exist and thus dominate the exit time.
The probability on any given time step t to flip a local leader i with degree k_i is formally
given by:

$$P_{flip}(k_i, t) = \sum_j S_{ij} \frac{k_j^{\alpha}}{k_i^{\alpha} + k_j^{\alpha}} P(\sigma_i \neq \sigma_j, t) \tag{31}$$

where $P(\sigma_i \neq \sigma_l, t)$ is the probability that the neighboring state differs from the local
leader's state at time t.

Even neglecting the probability of differing states, the probability of a local leader adopt-
ing the state of a neighboring node has interesting properties. Assuming independence of the
behavior of different local leaders in the same network, the mean time to take on a neigh-
boring state is easy to calculate for a given node. From simulations, the mean time to flip
of the most isolated local leader (who is not also the global leader) scales with system size
as shown in Fig. 5. While this does not have quantitative agreement with time to consen-
sus, the qualitative behavior is consistent. Local leaders see outside states more rarely under
node update than link update and the onset of slow flipping of local leaders starts earlier for
$\nu = 2.4$ than $\nu = 2.8$.

5 Summary

Recent work [11] has found an approximate mean exit time for a duplication process on
networks with arbitrary edge weights, assuming that diffusion is the dominant time scale.
Our model can be considered a weighted directed network where the weight of an edge from
i to j is given by W_{ij}^{α}, so it fits into the framework described. In this work, we demonstrate
that there are at least two natural ways for this estimate of exit time to fail. As previously
observed in Baxter et al., the time for the system to reach a metastable equilibrium can be
at least as large as the diffusive exit time scale. We see this in the edge selection process
for values of α and ν where the diffusive exit time vanishes as system size gets large. The
frustrated dynamics in the node selection mode, however, presents a new way in which the

Fig. 5 The scaling of mean time for the most isolated local leader to adopt the state of a neighbor, based on 1000 networks and $N = 1000$–32000. *Circles* are for $\nu = 2.4$, *squares* for $\nu = 2.8$, connecting lines indicate node update, markers only indicates edge update. Note that this ignores the probability that a neighbor has an opposite state, therefore certainly underestimating the time to flip, but still gives qualitative agreement to the behavior observed in simulations
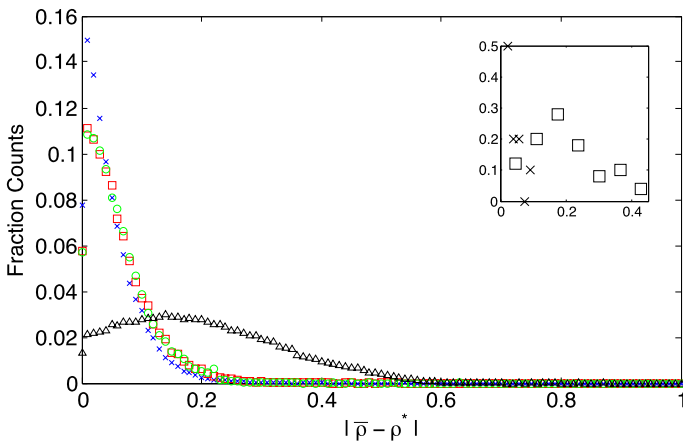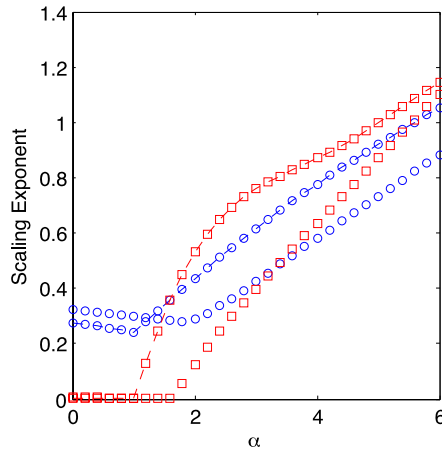
Fig. 6 Normalized histogram of the absolute value of the time average deviation from ensemble equilibrium value $\rho*$ per node for $N = 3000$, $\nu = 2.4$, and $\alpha = 1$ ($\times$), 1.5 ($\bigcirc$), 2 ($\square$), and 5 ($\triangle$) under node update dynamics, drawn over 50 (for $\alpha = 5$) or 10 (for all other α values) runs. For diffusive values of α, the deviations are gaussian and peaked around zero. For $\alpha = 5$, nodes do not center about $\rho*$. The inset shows a normalized histogram of mean deviation values per network for $\alpha = 1$ ($\times$) and 5 ($\square$). The diffusive case is peaked around zero, whereas the frustrated case is broad and nonzero, showing large differences across networks

diffusive estimate can fail. System dynamics are driven by a small number of topologically special nodes, breaking the assumption that a continuum description applies.

More important than the specific model we have presented is the lesson about processes on scale-free networks. Because random scale-free networks are well known to exhibit large deviations, small differences in stochastic processes performed on them can create large differences in resulting behavior. In our model, the seemingly modest transition between node selection and edge selection radically changed the scaling of consensus time with system size, with edge selection being able to reach consensus much faster than node selection for the same ensemble average magnetization. Moreover, slight changes in the role of degree
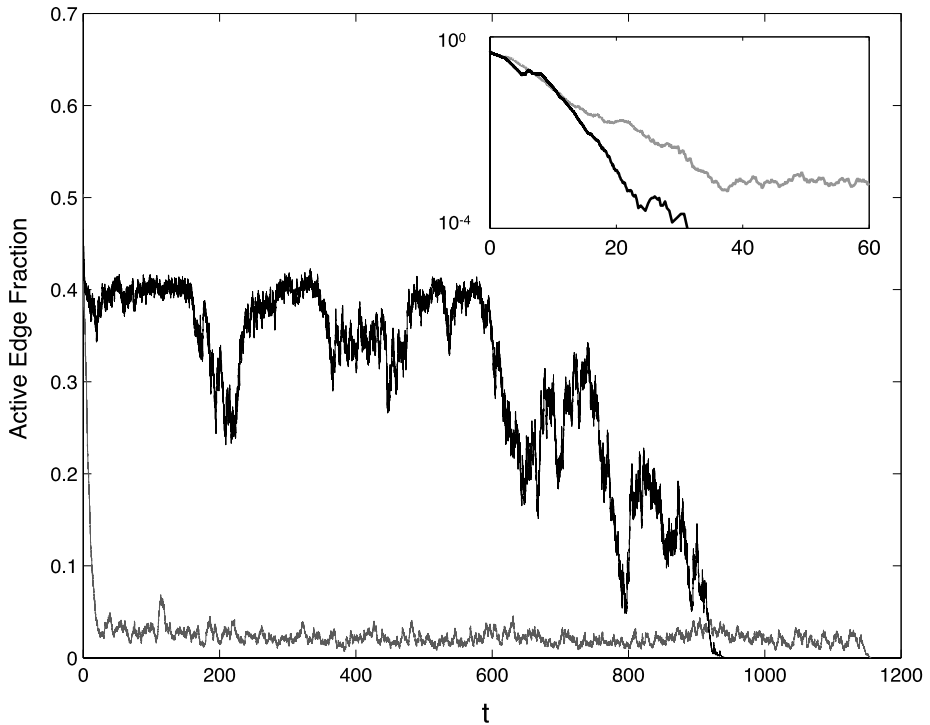
Fig. 7 Typical dynamics of the fraction of active edges (edges connecting nodes with different states) for diffusion and frustration. Both have $N = 10000$, $\nu = 2.8$, and edge selection. The *top line*, *in black*, is for $\alpha = 1$ and the *bottom line*, *in gray*, is for $\alpha = 8$. The $\alpha = 1$ case is diffusive and fluctuates to convergence after reaching the ensemble average value. The $\alpha = 8$ case decays exponentially to a value greater than the ensemble average value, because a locally highest degree node or cluster is slow to flip. Just before $t = 1200$, these nodes flip and the system reaches the absorbing state. Inset shows early time dynamics on a similar network for $\alpha = 10$ *in gray* and $\alpha = 4$ *in black*. Both decay exponentially initially, but the high α case becomes frustrated and the other continues to convergence

were able to induce a transition between a regime with an exit time dictated by global topology to one where it is dominated by particular local features of the network.

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